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Unified Formulation for Contact Melting Inside a Symmetric Enclosure

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The contact melting progress of phase-change material inside the symmetric enclosure with a continuous boundary is generally studied, and a unified treatment for the heat transfer is proposed. The mathematical expressions of the dimensionless film thickness, melting rate, dimensionless time to complete melting, and Nusselt number for the contact melting processes are derived generally, which can be used for the analysis of the contact melting inside different geometric enclosures. By applying the expressions to the analysis of contact melting inside the cylindrical and elliptical tubes, as well as the spherical capsule, the concrete methods and steps are given. It is found that some major results in the published literature are easily deduced and unified in this paper.

Nomenclature

Ar	=	Archimedes number
c_p	=	specific heat
E	=	perimeter
Fo	=	Fourier number
g	=	gravitation acceleration
H	=	height of the molten liquid
h	=	distance tangential
L	=	melting latent heat
L_m	=	modified melting latent heat
Nu	=	Nusselt number
P	=	pressure
Pr	=	Prandtl number
$\bar{q}$	=	average heat flux
R	=	radius of cylinder or sphere
R_D	=	modified radius
Ste	=	Stefan number
s	=	distance normal
T	=	temperature
T_e	=	initial solid temperature
T_m	=	melting point
T_w	=	wall temperature of the capsule
t	=	time
U	=	melting velocity of solid phase-change material at time t
u	=	tangential velocity
V	=	volume
V_s	=	volume of the solid at time t
V^*	=	dimensionless melting rate
x, y	=	abscissa, ordinate
x_0, y_0	=	coordinates of solid right position at time t
x_1, y_1	=	initial coordinates of solid top position
α	=	thermal diffusivity
δ	=	film thickness
λ	=	thermal conductivity
μ	=	dynamic viscosity
ν	=	kinematic viscosity
ρ	=	density
τ	=	dimensionless time
τ_f	=	dimensionless time to complete melting
ϕ	=	normal angular position

Superscript

$*$	=	dimensionless quantity

Subscripts

l	=	liquid state
s	=	solid state

I. Introduction

PHASE-CHANGE thermal storage devices have gained importance for many applications in which a large amount of energy must be transferred and stored, such as solar energy and heat pump air conditioning systems, because of their high energy density and isothermal behavior during charging and discharging. One very important solid–liquid phase-change process is contact melting inside an enclosure. Since Nicholas and Bayazitoglu [1] first studied contact melting as a distinct and important mechanism during melting inside a horizontal cylindrical tube, more works focused on different geometric heat sources and boundary conditions have been found in the literature [2–26]. For example, Bareiss and Beer [2], and Fomin and Saitoh [15] reported analytical, experimental, and numerical results for the contact melting inside a horizontal cylindrical tube, respectively. Moore and Bayazitoglu [4], Bahrami and Wang [5], and Roy and Sengupta [6] analyzed the contact melting inside a sphere. Dong et al. [7], Hirata et al. [8], and Chen et al. [9] investigated contact melting inside horizontal rectangular capsules. Chen et al. [10] also studied contact melting inside an elliptical tube, etc. In addition, some investigators [3,13] made unified analyses about spheres, horizontal cylindrical tubes, rectangle capsules, and vertical cylindrical tubes. But there are some problems in the reports, namely, either the force balance equation is singly listed as a complementary equation [3] without being analyzed and depicted in unification, or the unified analysis can only be applied to the heat resources with a given shape, such as sphere, horizontal cylindrical tube, rectangular capsule, and vertical cylindrical tube, by employing the different factor [13]. The results cannot be applied to contact melting under the condition of heat resources with other shapes. In the present work, the contact melting of phase-change material (PCM) inside a symmetric enclosure with a continuous boundary is generally analyzed. The theoretical formulas of the melting parameters are obtained. The results obtained in the published literature for the contact melting inside the cylindrical tube, sphere, rectangular capsule, and elliptical tube can be deduced from the formulas.

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II. Analysis

A. Contact Melting Inside a Horizontal Symmetric Capsule

Consider the contact melting of PCM inside a horizontal symmetric heating capsule formed by a continuous curve, within which the curvature radius is as shown in Fig. 1. The curve function is $y = f(x)$. The Cartesian coordinate origin is set up at the point of intersection of symmetric axis y and bottom tangent axis x, and the tangent and normal coordinates (h, s) are based on the curve. The PCM is initially in solid phase and entirely at the initial temperature T_e, and, during the melting process, the capsule wall temperature keeps at a constant and uniform T_w, larger than melting point T_m of PCM. As noted earlier, Bareiss and Beer [2], and Bahrami and Wang [5] considered contact melting inside the horizontal cylindrical tube and sphere. They concluded that melting at the upper solid surface was approximately 10–15% of the total melt, and so we assume that the convection and melting at the upper solid surface can be neglected. As summarized by Roy and Sengupta [6], and Bejan [25], the main features or assumptions of the analytical model are as follows:

1) The melting process is quasi steady, i.e., at any time, the weight of the solid PCM is balanced by the excess pressure built in the melted liquid layer.

2) The solid PCM descends vertically and axisymmetrically during the melting processes, as seen in the previous experiments of Bareiss and Beer [2], and Moore and Bayazitoglu [4], i.e., there is no slip at the capsule wall and melting front, and the shape of the upper surface of the solid PCM stays unchanged.

3) The melted liquid film at the contact surface is very thin and much smaller than the size of the capsule, so that heat transfer through the melted liquid film is by conduction in s direction.

4) The pressure at the two openings of the close contact gap is equal to the hydrostatic pressure in the upper pool of liquid.

5) The shear stress experienced by the solid PCM as it drops and sweeps the lateral wall is negligible, and no mushy region is formed.

6) The temperature in the thin liquid layer can be a linear distribution or a quadratic polynomial distribution in s direction; when the temperature difference between the solid PCM and wall or Ste is very small, there is no obvious difference between the two distributions.

Then, the governing equations and boundary conditions can be simplified as follows, respectively,

$$\mu_l \frac{\partial^2 u}{\partial s^2} = \frac{dP}{dh} \tag{1}$$

$$\frac{\partial^2 T}{\partial s^2} = 0 \tag{2}$$

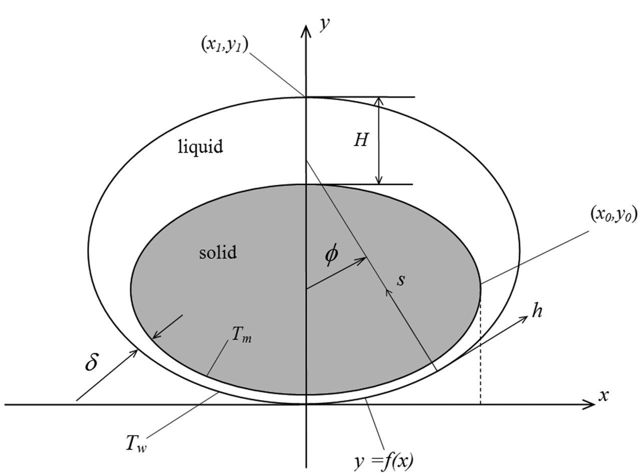

Fig. 1 Physical model and coordinates.

$$\lambda_l \frac{T_w - T_m}{\delta} = \rho_s U L_m \cos\phi \tag{3}$$

$$\int_0^\delta \rho_l u\,ds = \int_0^h \rho_s U \cos\phi\,\mathrm{d}h \tag{4}$$

$$\begin{aligned} u = 0, T = T_w, \quad &\text{at } s = 0 \\ u = 0, T = T_m, \quad &\text{at } s = \delta \end{aligned} \tag{5}$$

where U is the melting velocity of solid PCM at any time, which is equal to the derivative of the height of molten liquid to time. From Eqs. (1–5), the film thickness and the pressure in it can be obtained as follows:

$$\delta = \frac{\lambda_l (T_w - T_m)}{L_m \rho_s \cos\phi} \frac{\mathrm{d}t}{\mathrm{d}H} \tag{6}$$

$$P = -12 \frac{\mu_l \rho_s^4 L_m^3}{\rho_l^4 \alpha_l^3 C_p^3 (T_w - T_m)^3} \left(\frac{\mathrm{d}H}{\mathrm{d}t}\right)^4 \int_{x_0}^{x} \cos^2\phi x\,\mathrm{d}x \tag{7}$$

The force balance acting on the solid PCM is

$$2\int_0^{x_0} P\,\mathrm{d}x = g(\rho_s - \rho_l) V_s \tag{8}$$

where V_s is the volume of the solid PCM at any time t, which can be calculated by

$$V_s = 2\int_0^{y_0} x\,\mathrm{d}y + 2\int_{y_0}^{y_1 - H} \bar{x}\,\mathrm{d}y \tag{9}$$

where $\bar{x}$ satisfies the function $y = f(\bar{x}) - H$, and y_1 is the height of the solid at $t = 0$. Substituting Eqs. (7) and (9) into Eq. (8) results in

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \left\{ \frac{\rho_l^4 \alpha_l^3 C_p^3 (T_w - T_m)^3 g(\rho_s - \rho_l)(\int_0^{y_0} x\,\mathrm{d}y + \int_{y_0}^{y_1 - H} \bar{x}\,\mathrm{d}y)}{12\mu_l L_m^3 \rho_s^4 \int_0^{x_0} \int_x^{x_0} \{z/1 + [f'(z)]^2\}^{-1}\,\mathrm{d}z\,\mathrm{d}x} \right\}^{1/4} \tag{10}$$

where z is a variable. The dimensionless parameters are defined as follows:

$$H^* = \frac{H}{y_1}, \quad Ste = \frac{C_p(T_w - T_m)}{L_m}, \quad Fo = \frac{4\alpha_l t}{y_1^2}$$
$$Ar = (1-\rho^*)\frac{g y_1^3}{8\nu_l^2}, \quad \rho^* = \frac{\rho_l}{\rho_s}, \quad \bar{x}^* = \frac{\bar{x}}{y_1}$$
$$y^* = \frac{y}{y_1}, \quad \delta^* = \frac{\delta}{y_1}, \quad x^* = \frac{x}{y_1}, \quad z^* = \frac{z}{y_1}$$
$$x_0^* = \frac{x_0}{y_1}, \quad y_0^* = \frac{y_0}{y_1}, \quad \tau = Ste Fo$$

Then, Eq. (10) can be rewritten as follows:

$$\frac{\mathrm{d}H^*}{\mathrm{d}\tau} = 0.226 \left\{ \frac{\rho^{*3} Ar Pr (\int_0^{y_0^*} x^*\,\mathrm{d}y^* + \int_{y_0^*}^{1-H^*} \bar{x}^*\,\mathrm{d}y^*)}{Ste \int_0^{x_0^*} \int_{x^*}^{x_0^*} \{z^*/1 + [f'(z^*)]^2\}^{-1}\,\mathrm{d}z^*\,\mathrm{d}x^*} \right\}^{1/4} \tag{11}$$

Integrating Eq. (11) with initial condition $H^*|_{\tau=0} = 0$ yields

$$\tau = 4.43 \left(\frac{Ste}{Pr Ar \rho^{*3}}\right)^{1/4} \int_0^{H^*} \left\{ \int_0^{x_0^*} \int_{x^*}^{x_0^*} \frac{z^*}{1 + [f'(z^*)]^2}\,\mathrm{d}z^*\,\mathrm{d}x^* \right\}^{1/4} \times \left(\int_0^{y_0^*} x^*\,\mathrm{d}y^* + \int_{y_0^*}^{1-H^*} \bar{x}^*\,\mathrm{d}y^* \right)^{-1/4} \mathrm{d}H^* \tag{12}$$

From Eq. (6), the dimensionless film thickness can be obtained as follows:

$$\delta^* = \frac{\rho^*}{4\cos\phi}\frac{\mathrm{d}\tau}{\mathrm{d}H^*} \tag{13}$$

The melting rate is

$$V^* = 1 - \frac{\int_0^{y_0} x\,\mathrm{d}y + \int_{y_0}^{y_1-H} \bar{x}\,\mathrm{d}y}{\int_0^{y_1} x\,\mathrm{d}y} = 1 - \frac{\int_0^{y_0^*} x^*\,\mathrm{d}y^* + \int_{y_0^*}^{1-H^*} \bar{x}^*\,\mathrm{d}y^*}{\int_0^1 x^*\,\mathrm{d}y^*} \tag{14}$$

With the heat flux averaged with respect to the area of the heating capsule wall, an average Nusselt number can be defined as follows:

$$\overline{Nu} = \frac{[\bar{q}/(T_w - T_m)]R_D}{\lambda_l} = \frac{L_m \rho_s R_D}{E\lambda_l(T_w - T_m)}\left(-\frac{\mathrm{d}V_s}{\mathrm{d}t}\right) \tag{15}$$

where E and R_D are the perimeter and modified radius of the capsule, respectively, that is

$$E = 2\int_0^{y_1}\{1 + 1/[f'(x)]^2\}^{1/2}\,\mathrm{d}y + 2x_1$$

$$R_D = \left(4\int_0^{y_1} x\,\mathrm{d}y\right)\Big/E$$

Substituting E and R_D into Eq. (15) yields

$$\overline{Nu} = \frac{-2L_m\rho_s}{\lambda_l(T_w - T_m)}\frac{\int_0^{y_1} x\,\mathrm{d}y}{(\int_0^{y_1}\{1 + 1/[f'(x)]^2\}^{1/2}\,\mathrm{d}y + x_1)^2} \times \frac{\mathrm{d}(\int_0^{y_0} x\,\mathrm{d}y + \int_{y_0}^{y_1-H}\bar{x}\,\mathrm{d}y)}{\mathrm{d}t} \tag{16}$$

B. Contact Melting Inside an Axisymmetric Capsule

For the contact melting inside a symmetric heating capsule formed by the rotation of the continuous curve around axis y, an analysis that is analogous to the aforementioned method can be constructed. Equations (1–3) and (5) are valid here, but the mass balance in the liquid layer is

$$\int_0^{\delta}\rho_l u x d s = \int_0^h \rho_s U x\cos\phi\,\mathrm{d}h \tag{17}$$

Combining Eqs. (1–3), (5), and (17) results in

$$P = -6\frac{\mu_l\rho_s^4 L_m^3}{\rho_l^4\alpha_l^3 C_p^3(T_w - T_m)^3}\left(\frac{\mathrm{d}H}{\mathrm{d}t}\right)^4\int_{x_0}^x \cos^2\phi x\,\mathrm{d}x \tag{18}$$

The force balance acting on the solid PCM is

$$2\pi\int_0^{x_0} P x\,\mathrm{d}x = g(\rho_s - \rho_l)V_s \tag{19}$$

where V_s is the volume of the solid PCM at time t, which can be calculated by

$$V_s = \pi\left(\int_0^{y_0} x^2\,\mathrm{d}y + \int_{y_0}^{y_1-H}\bar{x}^2\,\mathrm{d}y\right) \tag{20}$$

Substituting Eqs. (18) and (20) into Eq. (19) and integrating it with initial condition $H^*|_{\tau=0} = 0$ yields

$$\tau = 4.43\left(\frac{Ste}{PrAr\rho^{*3}}\right)^{1/4}\int_0^{H^*}\left\{\int_0^{x_0^*}\int_{x^*}^{x_0^*}\frac{z^*}{1+[f'(z^*)]^2}\mathrm{d}z^* x^*\,\mathrm{d}x^*\right\}^{1/4} \times \left(\int_0^{y_0^*} x^{*2}\mathrm{d}y^* + \int_{y_0^*}^{1-H^*}\bar{x}^{*2}\mathrm{d}y^*\right)^{-1/4}\mathrm{d}H^* \tag{21}$$

From Eq. (6), the dimensionless layer thickness can be obtained as follows:

$$\delta^* = \frac{1.11}{\cos\phi}\left(\frac{Ste\rho^*}{PrAr}\right)^{1/4}\left\{\int_0^{x_0^*}\int_{x^*}^{x_0^*}\frac{z^*}{1 + [f'(z^*)]^2}\,\mathrm{d}z^* x^*\,\mathrm{d}x^*\right\}^{1/4} \times \left(\int_0^{y_0^*} x^{*2}\,\mathrm{d}y^* + \int_{y_0^*}^{1-H^*}\bar{x}^{*2}\,\mathrm{d}y^*\right)^{-1/4} \tag{22}$$

The melting rate is

$$V^* = 1 - \frac{\int_0^{y_0^*} x^{*2}\,\mathrm{d}y^* + \int_{y_0^*}^{1-H^*}\bar{x}^{*2}\,\mathrm{d}y^*}{\int_0^1 x^{*2}\,\mathrm{d}y^*} \tag{23}$$

The dimensionless time required to complete melting of solid PCM can be derived by substituting $H^* = 1$ into Eq. (21). Equations (12), (13), (16), and (21–23) are the unified theoretical formulas of the melting parameters, from which the new results for the contact melting inside a concrete symmetric enclosure, or the results obtained in the published literature for the contact melting inside the cylindrical tube, sphere, rectangular capsule, and elliptical tube can be deduced.

III. Application and Discussion

Application 1: It is assumed that the symmetric capsule is a horizontal cylindrical tube with radius R, whose configuration in ordinates (x, y) satisfies the following equation:

$$x^2 + (y - R)^2 = R^2 \tag{24}$$

The boundary conditions are

$$y_1 = 2R, \quad x_1 = 0, \quad y_0 = R - H/2$$
$$x_0 = \sqrt{R^2 - (H/2)^2}, \quad \text{and} \quad \bar{x}^2 + (y - R + H)^2 = R^2 \tag{25}$$

Then, we have

$$\int_0^{y_0^*} x^*\,\mathrm{d}y^* + \int_{y_0^*}^{1-H^*}\bar{x}^*\,\mathrm{d}y^* = 0.5\int_0^{y_0^*}\sqrt{1 - (2y^* - 1)^2}\,\mathrm{d}y^* + 0.5\int_{y_0^*}^{1-H^*}\sqrt{1 - (2y^* - 1 + 2H^*)^2}\,\mathrm{d}y^* = (\arccos H^* - H^*\sqrt{1 - H^{*2}})/4 \tag{26}$$

$$\int_0^{x_0^*}\int_{x^*}^{x_0^*}\frac{z^*}{1 + [f'(z^*)]^2}\,\mathrm{d}z^*\,\mathrm{d}x^* = \int_0^{x_*}\int_{x^*}^{x_0^*} z^*(1 - 4z^{*2})\,\mathrm{d}z^*\,\mathrm{d}x^* = x_0^{*3}/3 - 4x_0^{*5}/5 = \frac{\sqrt{1 - H^{*2}}}{40}\left(\frac{2}{3} + \frac{1}{3}H^{*2} - H^{*4}\right) \tag{27}$$

$$\int_0^1 x^*\,\mathrm{d}y^* = \pi/8 \tag{28}$$

$$\int_0^{y_1} x\,\mathrm{d}y = \pi R^2/2 \tag{29}$$

$$\frac{\mathrm{d}(\int_0^{y_0} x\,\mathrm{d}y + \int_{y_0}^{y_1-H}\bar{x}\,\mathrm{d}y)}{\mathrm{d}t} = -2R^2\sqrt{1 - H^{*2}}\frac{\mathrm{d}H^*}{\mathrm{d}t} \tag{30}$$

$$\int_0^{y_1}\{1 + 1/[f'(x)]^2\}^{1/2}\,\mathrm{d}y + x_1 = \int_0^{2R}\sqrt{\frac{R^2}{R^2 - (y - R)^2}}\,\mathrm{d}y = \pi R \tag{31}$$

Substituting Eqs. (26) and (27) into Eq. (11) yields

$$\frac{dH^*}{d\tau} = 0.226\left(\frac{PrAr\rho^{*3}}{Ste}\right)^{1/4}\left[\frac{\arccos H^* - H^*\sqrt{1-H^{*2}}}{4x_0^{*3} - 16x_0^{*5}/5}\right]^{1/4} \tag{32}$$

Substituting $x_0^* = 0.5\sqrt{1-H^{*2}}$ into Eq. (32) and expanding it in series with a vanishing error by a second-order polynomial, results in

$$\frac{dH^*}{d\tau} = 0.402\left(\frac{PrAr\rho^{*3}}{Ste}\right)^{1/4}[0.81 + 0.26H^* + 0.2H^{*2}]^{-1} \tag{33}$$

Integrating Eq. (33) yields

$$\tau = 2\left(\frac{Ste}{PrAr}\right)^{0.25}\rho^{*-0.75}(H^* + 0.161H^{*2} + 0.057H^{*3}) \tag{34}$$

Substituting $H^* = 1$ into Eq. (34), the dimensionless time to complete melting is obtained as follows:

$$\tau_f = 2.44\left(\frac{Ste}{PrAr}\right)^{0.25}\rho^{*-0.75} \tag{35}$$

Substituting Eqs. (29–31) and (33) into Eq. (16) yields

$$\overline{Nu} = 0.2\left(\frac{PrAr}{Ste\rho^*}\right)^{0.25}\frac{\sqrt{1-H^*}}{0.63 + 0.2H^* + 0.16H^{*2}} \tag{36}$$

Equations (33–36) are the main results obtained by Bareiss and Beer [2] on the condition that the convective heat transfer at the upper interface of solid PCM is neglected. In addition, the dimensionless film thickness and melting rate are derived by substituting Eq. (32) into Eq. (13) and substituting Eqs. (26) and (28) into Eq. (14), respectively, as follows:

$$\delta^* = \frac{\rho^*}{\cos\phi}\left(\frac{Ste}{PrAr\rho^{*3}}\right)^{1/4}[0.5 + 0.16H^* + 0.12H^{*2}] \tag{37}$$

$$V^* = 1 - \frac{2(\arccos H^* - H^*\sqrt{1-H^{*2}})}{\pi} \tag{38}$$

Application 2: When the symmetric capsule is a horizontal elliptical tube with radius b and a, which configuration in coordinates (x, y) satisfies the following equations:

$$\frac{x^2}{a^2} + \frac{(y-b)^2}{b^2} = 1 \quad \text{or} \quad f(x) = -b\left(1 - \frac{x^2}{a^2}\right)^{1/2} \tag{39}$$

The boundary conditions are

$$y_1 = 2b, \quad x_1 = 0, \quad y_0 = b - H/2$$
$$x_0 = a\sqrt{1-(H/2b)^2}, \quad \text{and} \quad \frac{\bar{x}^2}{a^2} + \frac{(y-b+H)^2}{b^2} = 1 \tag{40}$$

The dimensionless parameters are expressed as follows:

$$H^* = \frac{H}{2b}, \quad Fo = \frac{\alpha_l t}{b^2}, \quad Ar = (1-\rho^*)\frac{gb^3}{\nu_l^2}$$
$$\bar{x}^* = \frac{\bar{x}}{2b}, \quad y^* = \frac{y}{2b}, \quad \delta^* = \frac{\delta}{2b}, \quad x^* = \frac{x}{2b}$$
$$z^* = \frac{z}{2b}, \quad x_0^* = \frac{x_0}{2b}, \quad y_0^* = \frac{y_0}{2b}, \quad J = b/a$$

Then, we can derive

$$\frac{dH^*}{d\tau} = \left(\frac{Ar\rho^{*3}Pr}{192Ste}\right)^{1/4} \times \left[\frac{J^2(1-J^2)(\arccos H^* - H^*\sqrt{1-H^{*2}})}{[J^2/(1-J^2)](\sqrt{1-H^{*2}} - \{\ln[1+\sqrt{(1-J^2)(1-H^{*2})}] - \ln[1-\sqrt{(1-J^2)(1-H^{*2})}]\}/\sqrt{1-J^2}) - [(1-H^{*2})^{1.5}/3]}\right]^{1/4} \quad \text{for } J < 1 \tag{41}$$

$$\frac{dH^*}{d\tau} = \left(\frac{Ar\rho^{*3}Pr}{192Ste}\right)^{1/4}\left[\frac{J^2(J^2-1)(\arccos H^* - H^*\sqrt{1-H^{*2}})}{[J^2/(J^2-1)]\{\sqrt{1-H^{*2}} - [\arctan\sqrt{(J^2-1)(1-H^{*2})}/\sqrt{J^2-1}]\} - [(1-H^{*2})^{1.5}]/3}\right]^{1/4} \quad \text{for } J > 1 \tag{42}$$

Expanding and integrating Eqs. (41) and (42) in series with a vanishing error by a second-order polynomial, results in

$$\tau = 2.49\left(\frac{Ste}{PrArJ^2}\right)^{0.25}\rho^{*-0.75}\left[f_1(0) + \frac{1}{2}f_1'(0)H^* + \frac{1}{6}f_1''(0)H^{*2}\right]H^* \quad \text{for } J < 1 \tag{43}$$

$$\tau = 3.722\left[\frac{Ste}{J^2(J^2-1)PrAr}\right]^{0.25}\rho^{*-0.75}\left[f_2(0) + \frac{1}{2}f_2'(0)H^* + \frac{1}{6}f_2''(0)H^{*2}\right]H^* \quad \text{for } J > 1 \tag{44}$$

where

$$\left.\begin{array}{c} f_1(0) = \left(\frac{10-6J^2}{3\pi}\right)^{0.25}; f_1'(0) = f_1(0)/\pi \\ f_1''(0) = \frac{5[(5/3-J^2)-(1-J^2)(\pi/2)^2]}{4(5/3-J^2)^{3/4}(\pi/2)^{9/4}} \end{array}\right\} \tag{45}$$

$$\left.\begin{array}{c} f_2(0) = \left\{\frac{2J^2}{(J^2-1)\pi}\left[1 - \frac{\arctan\sqrt{J^2-1}}{\sqrt{J^2-1}}\right] - \frac{2}{3\pi}\right\}^{1/4} \\ f_2'(0) = f_2(0)/\pi \\ f_2''(0) = 5f_2(0)/\pi^2 \end{array}\right\} \tag{46}$$

Substituting $H^* = 1$ into Eqs. (43) and (44), the dimensionless time to complete melting is obtained as follows:

$$\tau_f = 2.49\left(\frac{Ste}{PrArJ^2}\right)^{0.25}\rho^{*-0.75}\left[f_1(0)+\frac{1}{2}f_1'(0)+\frac{1}{6}f_1''(0)\right] \quad \text{for } J<1 \tag{47}$$

$$\tau_f = 3.722\left[\frac{Ste}{J^2(J^2-1)PrAr}\right]^{0.25}\rho^{*-0.75}\left[f_2(0)+\frac{1}{2}f_2'(0)+\frac{1}{6}f_2''(0)\right] \quad \text{for } J>1 \tag{48}$$

The average Nusselt number, dimensionless film thickness and melting rate can also be derived by the same method, respectively, as follows:

$$\overline{Nu} = \frac{3.21\sqrt{1-H^{*2}}}{\pi[1.5(1+J)-\sqrt{J}]^2}\left(\frac{Ste\rho^*}{PrArJ^2}\right)^{-1/4}\Big/[f_1(0)+f_1'(0)H^*+0.5f_1''(0)H^{*2}] \quad \text{for } J<1 \tag{49}$$

$$\overline{Nu} = \frac{2.15[J^2(J^2-1)]^{1/4}}{\pi[1.5(1+J)-\sqrt{J}]^2}\left(\frac{Ste\rho^*}{PrAr}\right)^{-1/4}\Big/[f_2(0)+f_2'(0)H^*+0.5f_2''(0)H^{*2}] \quad \text{for } J>1 \tag{50}$$

$$\delta^* = \frac{1.245}{\cos\phi}\left(\frac{Ste\rho^*}{J^2PrAr}\right)^{0.25}[f_1(0)+f_1'(0)H^*+0.5f_1''(0)H^{*2}] \quad \text{for } J<1 \tag{51}$$

$$\delta^* = \frac{1.861}{\cos\phi}\left(\frac{Ste\rho^*}{J^2(J^2-1)PrAr}\right)^{0.25}[f_2(0)+f_2'(0)H^*+0.5f_2''(0)H^{*2}] \quad \text{for } J>1 \tag{52}$$

$$V^* = 1-\frac{2(\arccos H^* - H^*\sqrt{1-H^{*2}})}{\pi} \tag{53}$$

Whereas $a=b=R$, namely $J=1$, the elliptical tube changes into a cylindrical tube. Then, from Eqs. (43–53), we can also obtain Eqs. (34–38).

Figures 2 and 3 illustrate the variation of the melting rate V^* and dimensionless film thickness δ_0^* at $\phi=0$ with the dimensionless time τ, respectively. It is seen that V^* and δ_0^* are the monotonically increasing function of τ, both of them decreasing with the increase of ΔT. From Fig. 3, it can also be found that δ_0^* is 2 or 3 orders of magnitude smaller than 1 during the whole melting process, which just satisfies application 3. The variation of dimensionless height H^* of the molten liquid with the dimensionless time ratio τ/τ_f is plotted in Fig. 4 according to Eq. (12). The experiment by Bareiss and Beer [2] and the calculation by Fomin and Saitoh [15] showed the the relationship between H^* and τ/τ_f is independent of temperature difference, Archimedes number, Prandtl number, and Stefan number. Equation (12) and Fig. 4 reflect the same results. From Fig. 5, it is found that the heat transfer characterized by $\overline{Nu}$ is affected by the temperature difference ΔT, and it is at maximum at the beginning and drops to zero at the end of the melting process.

Application 3: It is assumed that the axisymmetric heating capsule is a sphere with radius R, which is formed by the rotation of the continuous curve $y=R\pm\sqrt{R^2-x^2}$ around symmetric axis y. Analogous to the previous section, we have

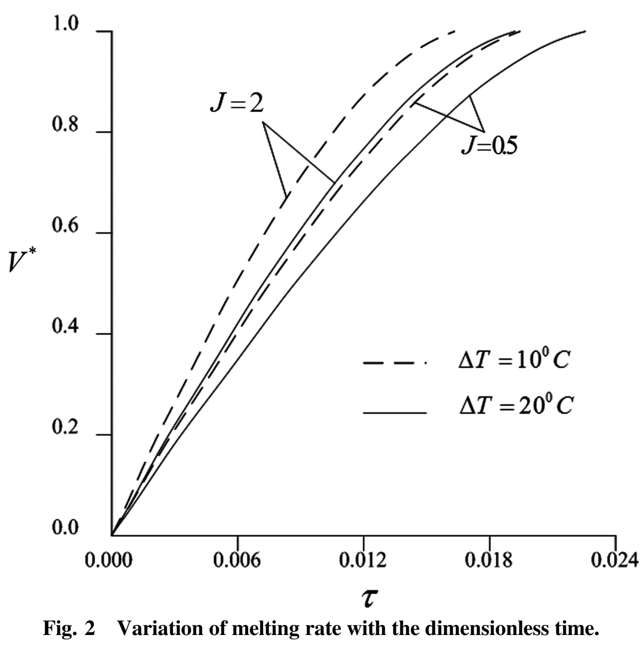

Fig. 2 Variation of melting rate with the dimensionless time.

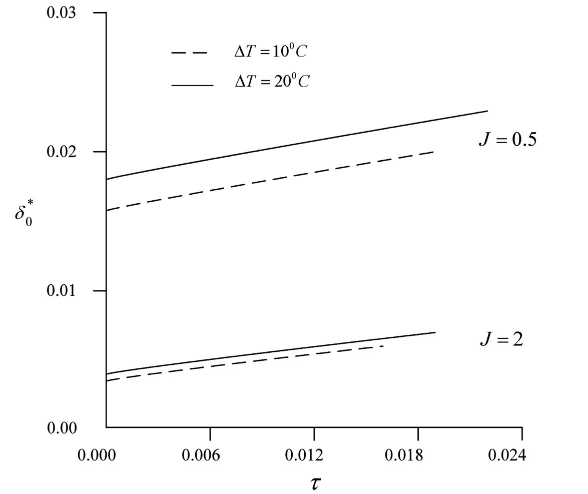

Fig. 3 Variation of dimensionless film thickness at the bottom with the dimensionless time.

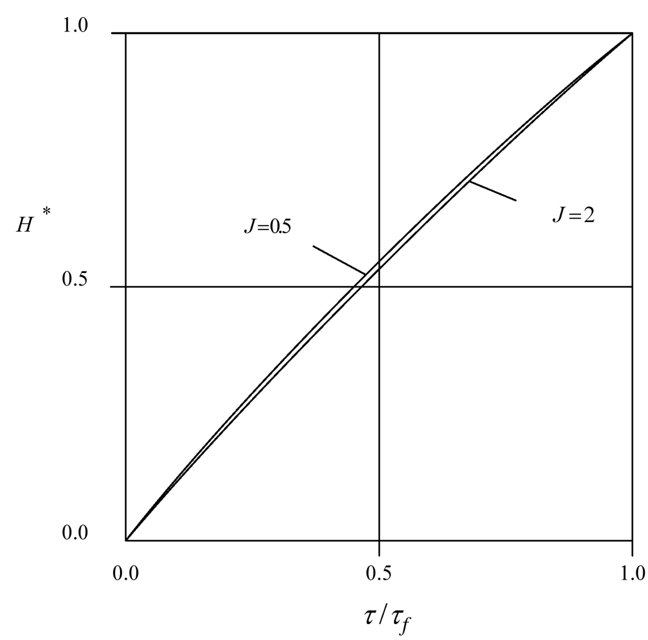

Fig. 4 Variation of dimensionless height of the molten liquid with the dimensionless time ratio.

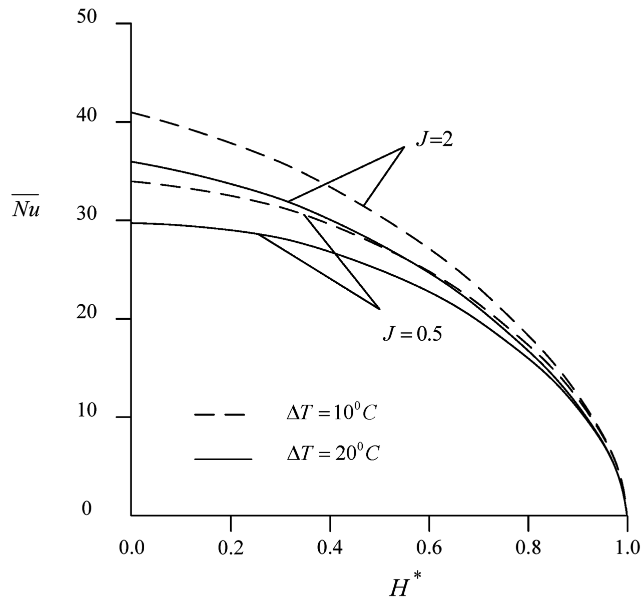

Fig. 5 Variation of average Nusselt number with the dimensionless height of the molten liquid.

$$\int_0^{y_0^*} x^{*2}\,dy^* + \int_{y_0^*}^{1-H^*} \bar{x}^{*2}\,dy^* = H^{*3}/12 - H^*/4 + 1/6 \quad (54)$$

$$\int_0^{x_0^*}\int_{x^*}^{x_0^*} \frac{z^*}{1+[f'(z^*)]^2}\,dz^* x^*\,dx^* = x_0^{*4}/8 - x_0^{*6}/3 = (1-3H^{*4}+2H^{*6})/384 \quad (55)$$

$$\int_0^1 x^{*2}\,dy^* = 1/6 \quad (56)$$

Substituting Eqs. (54) and (55) into Eq. (21) yields

$$\frac{dH^*}{d\tau} = \left(\frac{Pr\overline{Ar}}{12Ste}\right)^{1/4} \rho^* \left[\frac{2-3H^*+H^{*3}}{1-3H^{*4}+2H^{*6}}\right]^{1/4} \quad (57)$$

Here, $\overline{Ar}$ is defined as

$$\overline{Ar} = (1-\rho^*)\frac{g(2R)^3}{8\rho^*\nu_l^2} = Ar/\rho^*$$

The transcendental function within the square bracket of Eq. (57) can be substituted with a vanishing error by a third-order polynomial. Then, Eq. (57) can be derived as follows:

$$\frac{dH^*}{d\tau} = \left(\frac{Pr\overline{Ar}}{12Ste}\right)^{1/4} \rho^*[0.841 + 0.3H^* + 0.42H^{*2} - 0.147H^{*3}]^{-1} \quad (58)$$

Integrating Eq. (58) yields

$$\tau = 2\left(\frac{Ste}{Pr\overline{Ar}}\right)^{1/4} \rho^{*-1}(1.56H^* + 0.279H^{*2} + 0.261H^{*3} - 0.0686H^{*4}) \quad (59)$$

Substituting $H^* = 1$ into Eq. (59), the dimensionless time to complete melting is obtained as follows:

$$\tau_f = 2.03\left(\frac{\rho_s}{\rho_l}\right)\left(\frac{Ste}{Pr\overline{Ar}}\right)^{1/4} \quad (60)$$

Equations (58–60) are the main results for the contact melting inside a sphere obtained by Bahrami and Wang [5], and Fomin and Saitoh [15] (for $a = 0$). In addition, substituting Eqs. (54–56) into Eqs. (22) and (23), respectively, results in

$$\delta^* = \frac{1}{\cos\phi}\left(\frac{Ste}{Pr\overline{Ar}}\right)^{1/4}[0.39 + 0.14H^* + 0.2H^{*2} - 0.07H^{*3}] \quad (61)$$

$$V^* = (3H^* - H^{*3})/2 \quad (62)$$

For the contact melting inside the rectangular capsule [8] and vertical cylindrical tube [11] formed by the simple function $y = f(x)$, the related results can be easily derived by the same method as previously mentioned, therefore we do not give the unnecessary details.

IV. Conclusions

For the contact melting inside the capsules, the solid figuration, position, melting rate, melting time, and liquid film thickness are the basic characteristic parameters which described the phenomena and behavior of melting. In the present paper, a unified treatment of the contact melting of phase-change material (PCM) inside the symmetric enclosure is proposed, and the fundamental equations and mathematical expressions of the characteristic parameters for the contact melting processes are obtained. Of course, for simplification, some side features of the process such as convection are neglected. The application in Sec. III gives examples of solving the problems and validates the results obtained. It is shown that whether the treatment of melting is difficult or easy depends on the curve function $y = f(x)$. For example, the results for contact melting inside the rectangular capsule and vertical cylindrical tube are easily derived. But for the contact melting inside the symmetric enclosures with complex curve function $y = f(x)$, to obtain the characteristic parameters may need numerical calculation.

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